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# Application of Chaotic-Motion Motors to Industrial Mixing Processes

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**Abstract**—Industrial mixers are among the most expensive and ineffective equipment in food, drug, chemical and semiconductor industries. Effects of this ineffectiveness are not only energy wastage, but also can be disastrous. Instead of using complicated mechanical means, this paper presents a new way which can electrically produce the desired chaotic motion for industrial mixing processes. Namely, chaoization of the DC motor (the agitator) using time-delay feedback control is proposed and implemented. Theoretical derivation and computer simulation are provided to illustrate the controllable chaotic motion. Moreover, the mixing effectiveness is experimentally verified by an acid-base neutralization reaction.

**Keywords**—chaotic; mixing; motors

## I. INTRODUCTION

Stirred tanks are one of the most important mixing devices in food, drug, chemical and semiconductor industries. They can be used for both continuous and batch processes, and can process single liquid phase as well as liquid-liquid, gas-liquid and solid-liquid dispersions. One of the major limitations of stirred tanks is segregated regions formed in open impellers due to low Reynolds number where turbulent mixing cannot be achieved [1]. The persistence of these segregated regions requires more energy consumption or higher rotational speed. As a result, industrial mixers are among the most expensive and ineffective equipment. The industrialists and academics in the USA have estimated that the cost of ineffective industrial mixing is of the order of US\$ 1 to 10 billion per annum [2]. Effects of this ineffectiveness are not only energy wastage [3], but also can be disastrous (a nuclear-chemical waste explosion in Russia has been attributed to inadequate mixing of volatile compounds) [4]. While mixing can be improved by increasing the rate of stirring, such an approach is sometimes impractical. The product of many shear-sensitive materials, such as proteins and other macromolecules, are damaged in high shear rates in fast stirring in biotechnological applications [1]. Thus, the improvement of mixing is highly desirable and justifiable.

In recent years, chaotic mixing has been proposed to improve the energy efficiency and the degree of homogeneity by using mechanical means [2] – [4]. These mechanical means are essentially based on the design of impeller vanes to produce a practical chaotic motion. Instead of using complicated

impeller vanes, our idea is to electrically produce the desired chaotic motion to improve low Reynolds number mixing in a stirred tank.

Starting from the 1990's, a number of research activities on chaos in motors have been carried out. Most of them are based on the identification of chaos [5], the avoidance of chaos [6] and the stabilization of chaos [7] in various types of electric motors. Rather than negatively avoiding the occurrence of chaos in motors, a positive idea is to utilize the chaotic motion for some niche applications. Therefore, the purpose of this paper is to newly chaoize a motor, hence resulting in a controllable chaotic motion, for application to industrial mixing processes. Compared with those mechanical means, the proposed chaotic-motion motor not only produces the desired chaotic mixing, but also offers the advantages of high flexibility and high controllability. The key is to newly propose and implement the chaoization of the DC motor (the agitator) using time-delay feedback control [8].

There are many methods to evaluate the mixing processes, which can be divided into two categories: intrusive and non-intrusive. The intrusive methods include a probe or tracer put in the stirred tank to measure flow velocities and perturb the flow patterns that the investigators intend to measure. The non-intrusive methods, such as the Laser Doppler anemometer [9] and the acid-base neutralization reaction [10] are more attractive since they will not disturb the flow patterns. Increasingly, the acid-base neutralization reaction takes the advantages of simple arrangement and low cost.

In Section II, theoretical derivation will be provided, and in Section III computer simulation will be presented to illustrate the proposed chaotic-motion motor. Then, the implementation of the mixing system will be introduced in Section IV. In Section V, the results of chaotic mixing and constant speed mixing at the same power level will be compared to verify the effectiveness of the proposed chaotic mixing. Finally, conclusions will be drawn in Section VI.

## II. CHAOTIC-MOTION MOTOR

In this paper, a permanent magnet DC motor is used as the agitator, which can be modeled as:

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$$\frac{d}{dt} \begin{pmatrix} \omega(t) \\ i(t) \end{pmatrix} = \begin{pmatrix} -\frac{B}{J} & \frac{K_T}{J} \\ -\frac{K_E}{L} & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} \omega(t) \\ i(t) \end{pmatrix} + \begin{pmatrix} -\frac{T_l}{J} \\ \frac{V_{in}}{L} \end{pmatrix} \quad (1)$$

$$T = K_T i(t) \quad (2)$$

where  $B$  is the viscous damping coefficient,  $J$  is the load inertia,  $K_E$  is the back-EMF constant,  $K_T$  is the torque constant,  $L$  is the armature inductance,  $R$  is the armature resistance,  $T$  is the motor torque,  $T_l$  is the load torque,  $V_{in}$  is the DC supply voltage,  $\omega(t)$  is the motor speed and  $i(t)$  is the armature current. Fig. 1 shows the block diagram of the proposed control system for chaotic motion of the DC motor. The key is to employ time-delay feedback control based on the feedback law:

$$T = \mu \xi B f\left(\frac{\omega(t-\tau)}{\xi}\right) \quad (3)$$

where  $\mu$  is the torque parameter,  $\xi$  is the speed parameter,  $\tau$  is the time-delay parameter, and  $f(\cdot)$  is an integrable bounded function. Notice that all three parameters are adjustable to achieve the desired chaotic motion. The bounded function  $f(\cdot)$  satisfies the condition that the required torque will not be higher than the motor torque capability. In this paper,  $f(\cdot)$  is chosen as a sine function. In case there is no load torque, (1) can be rewritten as:

$$\frac{d\omega(t)}{dt} = -\frac{B}{J} \omega(t) + \frac{\mu \xi B}{J} \sin\left(\frac{\omega(t-\tau)}{\xi}\right). \quad (4)$$

### III. SIMULATION RESULTS

In order to conduct computer simulation, realistic system parameters are adopted. Table I summarizes some practical data of the DC motor. Making use of (4), speed bifurcation diagrams with respect to various adjustable parameters are readily deduced. These bifurcation diagrams can illustrate how the system behavior is affected by varying the parameters at a glance. When selecting  $\xi = 10$  and  $\tau = 1.5$  s, the speed bifurcation diagram with respect to  $\mu$  is shown in Fig. 2. It can be seen that the motor initially operates at a fixed point (which is equivalent to normal or so-called the period-1 operation) with a small value of  $\mu$ . Fig. 3 shows the corresponding motor speed and armature current when  $\mu = 1.4$ . With the increase of  $\mu$ , the motor bifurcates to the period-2 operation which is equivalent to an abnormal subharmonic operation. Fig. 4 shows the corresponding speed and current when  $\mu = 2.5$ . Finally, the motor exhibits chaotic motion when  $\mu$  is further increased. Fig. 5 shows the corresponding speed and current when  $\mu = 5.0$ . When the motor is run in the chaotic mode, both the amplitude and direction of the motor speed and armature current change with time and presents ergodicity in the range illustrated in Fig. 2. It is this character of ergodicity that differs chaotic mixing from normal constant speed mixing.

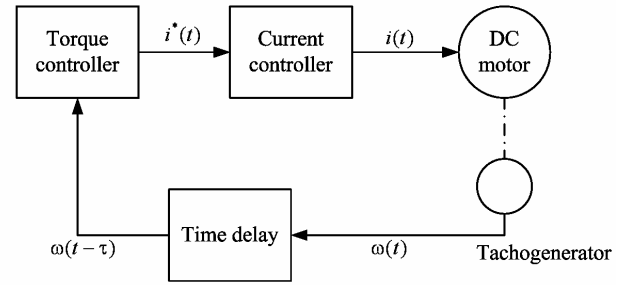


Figure 1. Control system.

TABLE I. MOTOR PARAMETERS

$B$	0.0022 Nm/(rad/s)
$J$	0.000010388 Nm/(rad/s <sup>2</sup> )
$K_E$	0.04975 V/(rad/s)
$K_T$	0.04998 Nm/A
$L$	0.4 mH
$R$	1.1 $\Omega$
$V_{in}$	24 V

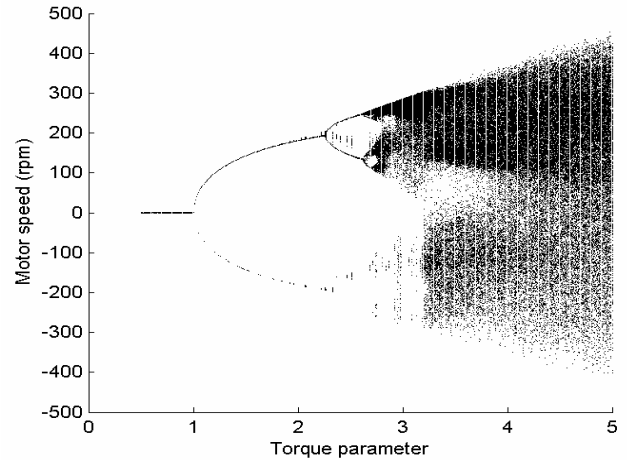


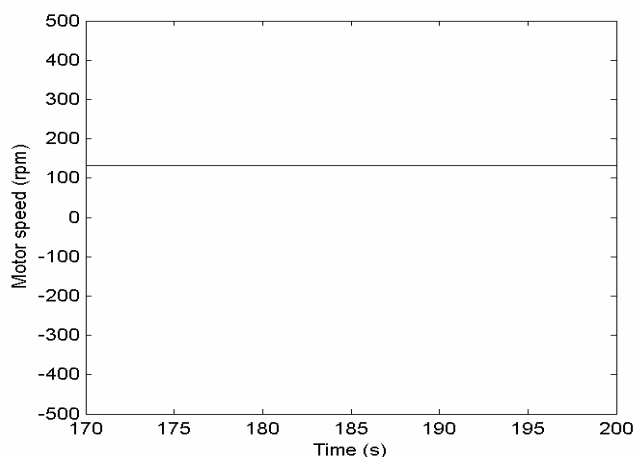
Figure 2. Speed bifurcation diagram with respect to torque parameter.

Torque parameter  $\mu$  is found to be the bifurcation parameter. Next, we will investigate how the other two adjustable positive parameters  $\xi$  and  $\tau$  will affect the chaotic performance of the motor. When selecting  $\mu = 5$  and  $\tau = 1.5$  s, the speed range with respect to  $\xi$  is shown in Fig. 6. The corresponding chaotic speed and current waveforms when  $\xi = 20$  is shown in Fig. 7. It can be seen that  $\xi$  can be used to adjust the speed range of the chaotic motion, and that is why it is named speed parameter. Furthermore, when selecting  $\mu = 5$  and  $\xi = 10$ , the corresponding speed range with respect to  $\tau$  is

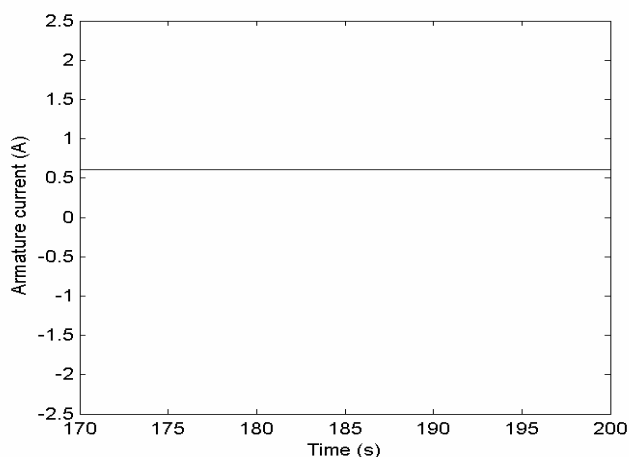
shown in Fig. 8. The corresponding chaotic speed and current waveforms when  $\tau=0.01$  s is shown in Fig. 9. It can be seen that the change of time-delay constant has little effect on the speed range. It should be noted that the significance of this parameter lies in the realization of the system. If the parameter is too large, the refresh rate of the control system is too low; while the parameter is too small, it requires too much computer resource.

#### IV. IMPLEMENTATION

The mixing apparatus consists of a tank and an impeller spun by a digitally-controlled drive mounted vertically on a stand with its shaft positioned at the center of the tank. The shaft is mounted through a holding plate, which ensures consistent positioning between experiments and minimizes oscillations of the shaft tip. The tank is a one liter glass beaker. The agitator is a SANYO R406 DC servo motor. The stand is a BOSCH BS 35 drill holder. The whole set-up is shown in Fig. 10.

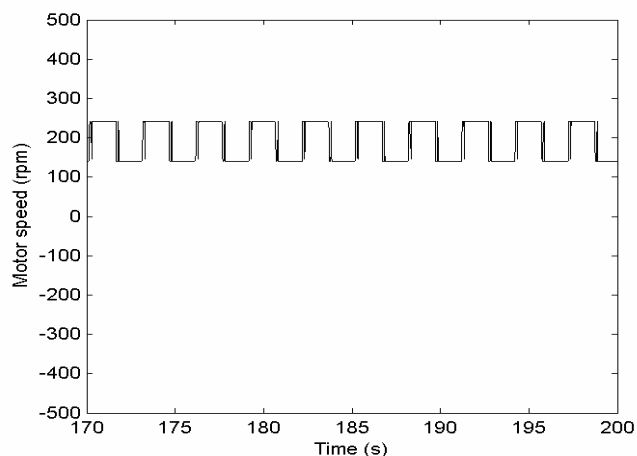


(a)

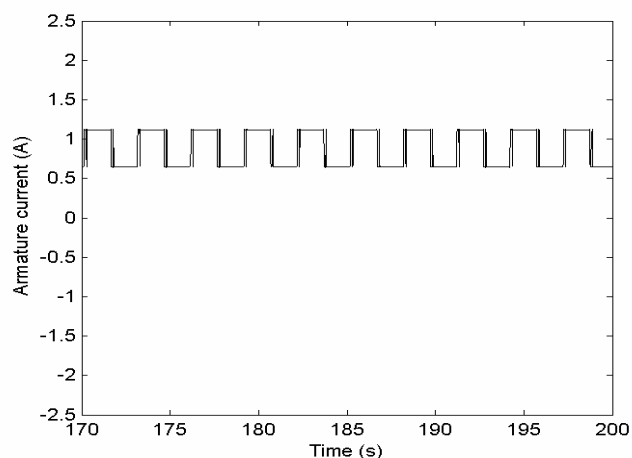


(b)

Figure 3. Period-1 operation with respect to torque parameter. (a) Motor speed. (b) Armature current.



(a)



(b)

Figure 4. Period-2 operation with respect to torque parameter. (a) Motor speed. (b) Armature current.

It should be noted that the mixture with low or moderate Reynolds number are particularly difficult to achieve effective mixing since the corresponding flow is laminar, whereas the mixture with high Reynolds number can easily achieve effective or so-called turbulent mixing. Thus, in this paper, a low-Reynolds-number mixture (light corn syrup) is purposely adopted so that the mixing effectiveness can be evaluated.

Firstly, the tank is filled with 200 ml light corn syrup, 5 ml pH indicator solution (universal indicator) and 5 ml solution of 1 N HCl. The system is mixed until a uniform red color is observed since the solution is acidic. Next, a solution of well mixed solution (in dark green) of 100 ml light corn syrup, 2.5 ml pH indicator solution and 2.5 ml 1 N NaOH is added into the tank. Although the solution is acidic as there is twice as much acid as base, there are dark green regions due to diffusion limitations caused by the highly viscous solvent (light corn syrup).

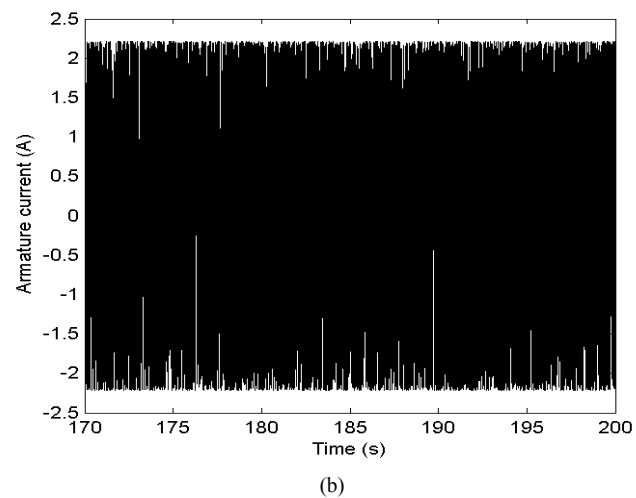
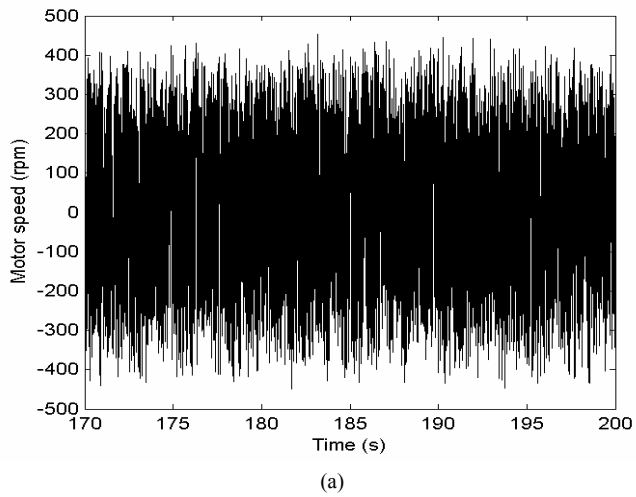


Figure 5. Chaotic operation with respect to torque parameter. (a) Motor speed. (b) Armature current.

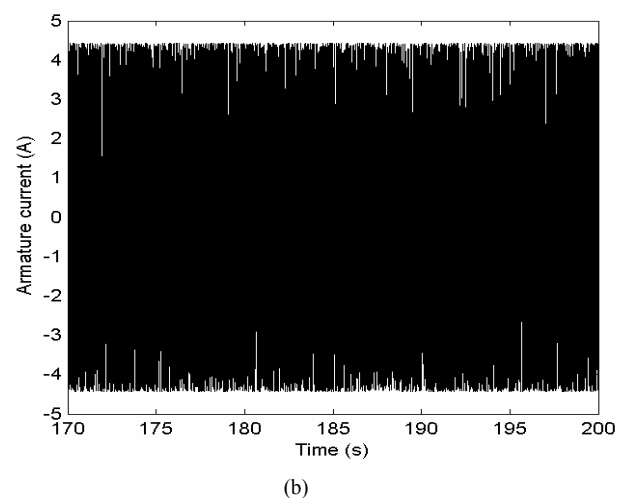
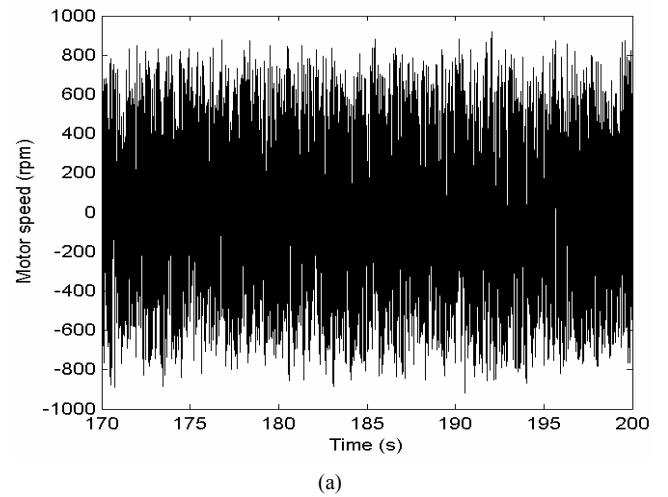


Figure 7. Chaotic operation with respect to speed parameter. (a) Motor speed. (b) Armature current.

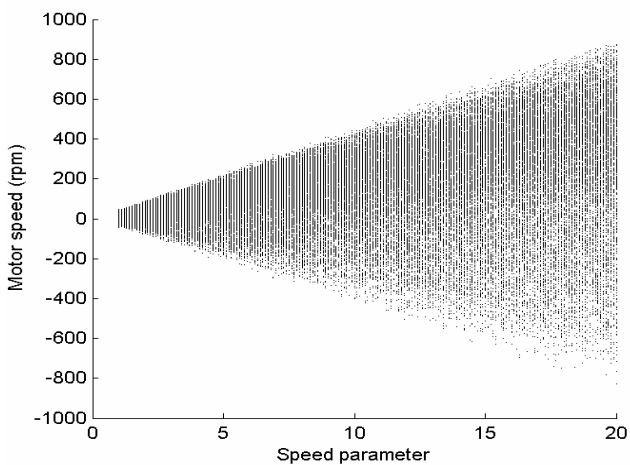


Figure 6. Speed range with respect to speed parameter.

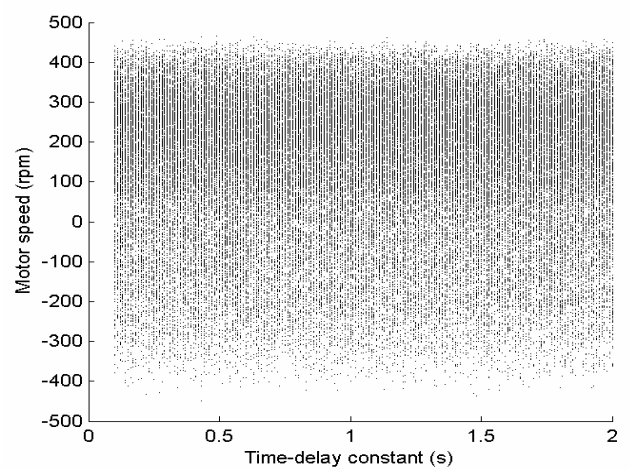


Figure 8. Speed range with respect to time-delay constant.

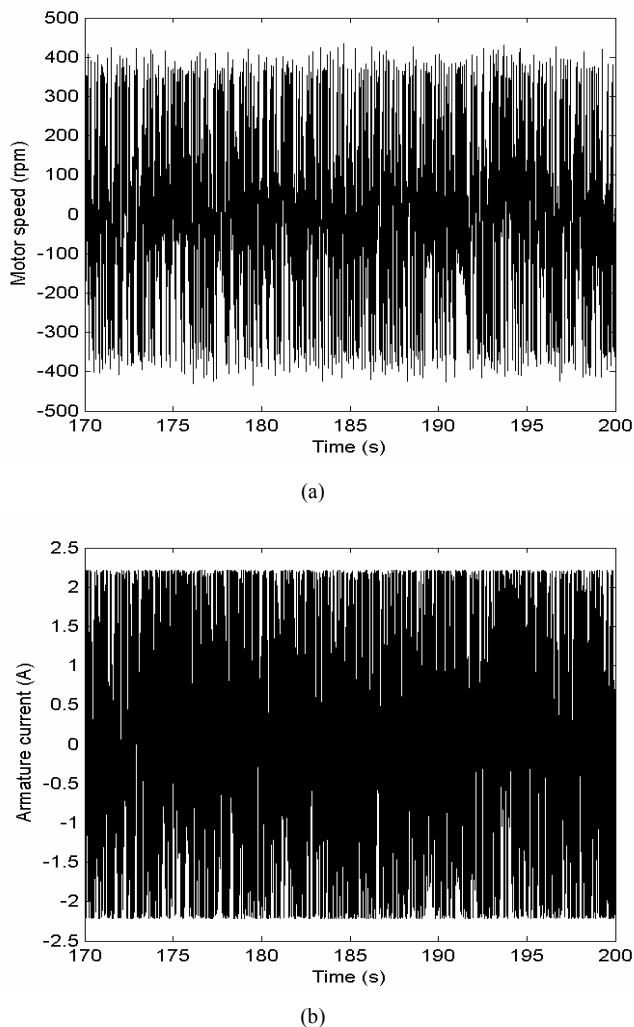


Figure 9. Chaotic operation with respect to time-delay constant. (a) Motor speed. (b) Armature current.

The beginning of the mixing of the acid-base solution is set as the zero of the time axis. The mixer is controlled by the dSPACE digital controller to perform chaotic mixing or normal constant speed mixing. The experiments are recorded using a web camera focused at the impeller.

## V. EXPERIMENTAL RESULTS

To assess whether chaotic mixing is more effective than normal constant speed mixing, it is equivalent to evaluate whether chaotic mixing can offer a more homogenous mixture under the same amount of energy consumption. If homogeneity is achieved in both cases, chaotic mixing consumes less energy. The experiment is designed to compare the mixing time needed to achieve homogeneity using the same average power. First, the chaotic mixing experiment is conducted. The armature voltage and current are measured and the power is integrated to calculate the energy consumed to achieve homogeneity. Then, the average power is calculated. The constant speed mixing is conducted under the same power.

Fig. 11 shows the system coloration during chaotic mixing. The corresponding chaotic speed is bounded between  $-1000$  rpm and  $1000$  rpm. Since twice as much acid as base is used in the experiment, the mixed regions of the stirred tank contain excessive acid, causing the indicator in these regions to appear red. On the other hand, the unreacted regions contain base and display a green color. Fig. 11(a) is taken after 10 s of mixing. Both the amplitude and direction of the speed of impeller change according to time. A segregated region, which exists in constant speed mixing, is not visible in chaotic mixing. Fig. 11 (b) and Fig. 11(c) show the system coloration after 30 s and 70 s of chaotic mixing, respectively. The green color is less and less. After 170 s, it reaches a uniform red color which is shown in Fig. 11(d). The average power consumed is found to be 4.2 W. It can also be found that the solution is transparent throughout the whole mixing process.

Fig. 12 shows the results of constant speed mixing which adopts the speed of  $1000$  rpm with the same power consumption of 4.2 W. In Fig. 12(a), the system has been stirred for 10 s. It can be seen that there is a segregated region above the impeller. Fig. 12(b) shows the coloration after 30 s, which indicates that the size of the segregated region is decreasing. After 70 s of mixing, the regions still remain segregated, which is shown in Fig. 12(c). Another feature of Fig. 12(c) is that the solution is less transparent compared with Fig. 12(b). It is due to the large amount of air bubbles generated during the mixing process. Fig. 12(d) shows the system after 170 s of mixing, which shows that the segregated region is still visible in the air-bubbled tank. It should be noted that this segregated region remains visible even after 300 s.



Figure 10. Experimental set-up.



(a)



(b)



(c)



(d)



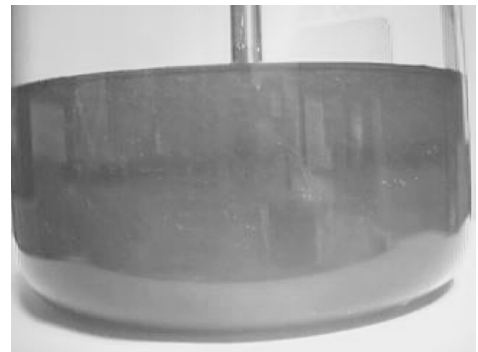
(a)



(b)



(c)



(d)

Figure 11. Coloration during chaotic mixing. (a) After 10 s. (b) After 30 s. (c) After 70 s. (d) After 170 s.

Figure 12. Coloration during normal constant speed mixing. (a) After 10 s. (b) After 30 s. (c) After 70 s. (d) After 170 s.

It can be seen that the segregated region prevents efficient mixing in the constant speed case. Reference [4] shows that the size and location of this region depend on the Reynolds number. Our experiment demonstrated that a large amount of energy needs to be consumed to destroy the segregated region. It also shows that chaotic mixing effectively prevents the formation of such segregated regions, leading to effective and fast mixing under low Reynolds number.

## VI. CONCLUSIONS

In this paper, chaotic mixing is electrically implemented using time-delay method on a DC motor, which acts as the agitator. Experimental results demonstrate that chaotic mixing prevents the formation of segregated regions, thus leading to efficient mixing compared with normal constant speed mixing.

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